

Critical velocities in the flow of liquid helium II between rotating cylinders

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(Received 17 October 1973, revised 8 May 1974)

The critical velocities in the flow of liquid helium II between coaxial rotating cylinders measured by Devaraj & Hollis Hallett (1972) are compared with those of an earlier investigation by Heikkilä & Hollis Hallett (1955), in view of the fact that the two sets of results are significantly different.

Heikkilä & Hollis Hallett (1955) and Woods (1957, 1958) studied the flow of liquid helium II between two coaxial cylinders, where the inner cylinder is stationary and the outer cylinder is rotating. They observed that the measured viscous torque on the inner cylinder is directly proportional to the velocity of rotation of the outer cylinder, and the flow is laminar for velocities upto about 0.1 cm sec^{-1} . For velocities higher than that, the observed torques were larger than the values expected from the linear law of laminar flow. These observations suggested the existence of a critical velocity V_c which marks the transition from the linear flow to a non-linear one. The critical velocity measured varied with the temperature of liquid helium II.

The Reynolds numbers corresponding to V_c were only of the order of 100 and were too small to attribute the sudden change in the behaviour of the flow to the onset of turbulence. The turbulence can only set in at Reynolds numbers of the order of 2000 shown by Taylor (1923, 1936). However, further experiments performed by Woods (1957) with ordinary liquids like carbon disulphide and acetone in the same apparatus showed that the critical velocity V_c existed even in these cases and the corresponding critical Reynolds numbers were also of the order of 100. On this basis it was suggested by Woods that the existence of the critical velocity V_c might be due to some form of secondary flow arising at the ends of the cylinders.

The critical velocity measurements of Heikkilä & Hollis Hallett and Woods were all made with one and the same apparatus. But, Devaraj & Hollis Hallett (1972) have made similar measurements of critical velocities in liquid helium II using a coaxial rotating cylinder system which had the relevant dimensions, like length and diameter of the cylinders, very much different from those of Heikkilä

et al. The values of critical velocities in the flow of liquid helium II between rotating cylinders obtained at various temperatures both by Heikkila & Hollis Hallett and by Devaraj & Hollis Hallett are given in table 1 and are also shown as a function of temperature in figure 1. The relevant dimensions of the rotating

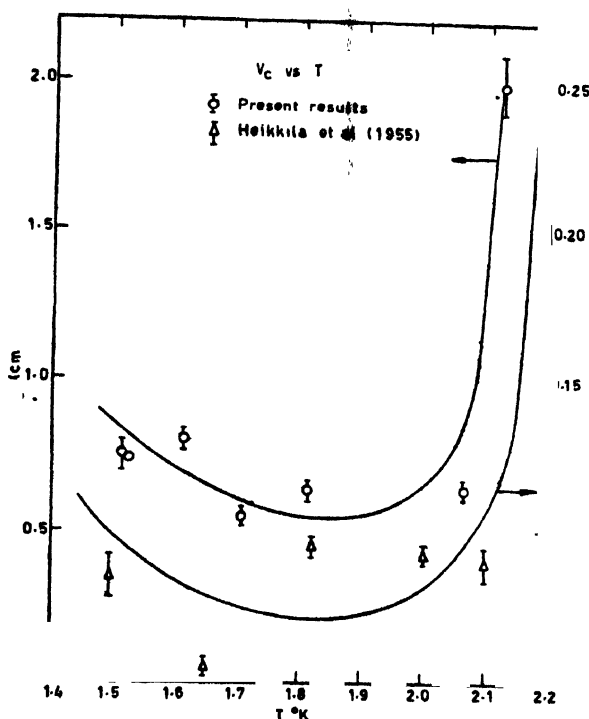


Fig. 1. Graph of critical velocity against temperature.

cylinders in the two cases are also indicated in table 1 for the purpose of comparison.

Table 1

Results of Heikkila <i>et al.</i>		Results of Devaraj <i>et al.</i>	
$d = 0.106$ cm. $(l/r) = 1.502$		$d = 0.0754$ cm. $(l/r) = 3.125$	
$l = 2.990$ cm. $r = 1.9913$ cm		$l = 2.5390$ cm. $r = 0.8126$ cm.	
T°K	V_c cm sec ⁻¹	T°K	V_c cm sec ⁻¹
2.180	0.25	2.117	2.0
2.100	0.09	2.068	0.64
2.002	0.093	1.814	0.63
1.820	0.095	1.706	0.55
1.650	0.055	1.610	0.80
1.496	0.085	1.525	0.74
		1.515	0.75

l = length of the inner cylinder, r = radius of the inner cylinder and d = gap-width between the inner and the outer cylinders.

It may be observed from figure 1 that in both the cases the critical velocities vary with temperature in very much the same way. This variation, in fact, is very similar to the variation with temperature of the viscosity of liquid helium II. This is easily understandable because the effect of viscosity, in general, is to increase the stability of the flow (Chandrasekhar & Donnelly 1957, Lin 1955) so that greater the viscosity of the fluid larger will be the critical velocity.

Although, in general, the results in the two cases show similar variation with temperature, the results of Devaraj *et al* are higher than those of Heikkilä *et al* by a factor of about eight consistently over the entire range of temperature studied. In the case of the rotating system of Devaraj *et al*, when compared with the other system, possibly, the increase in the length-to-radius ratio of the inner cylinder by a factor of about two has resulted in the observed increase of the critical velocity. There will, of course, also be a slight increase in the critical velocity due to the decrease in the gap-width between the cylinders.

The above results clearly indicate that the critical velocity does depend on the dimensions of the rotating cylinders. The critical velocity, probably, arises on account of some secondary flow setting in at the ends of the cylinders—the major source being the bottom end of the rotating cylinder, because of the additional torque it transmits to the stationary inner cylinder. The onset of any such secondary flow and therefore the critical velocity should depend on the dimensions of the cylinders, such as, the length-to-radius ratio of the inner cylinder and the gap-width between the coaxial cylinders.

ACKNOWLEDGMENT

The author expresses his grateful thanks to Professor A. C. Hollis Hallett, Department of Physics, University of Toronto for many useful discussions.

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